Vehicle Dynamics
(Longitudinal)

Mr.N.Rajmohan

EQUATION OF MOTION AND MAXIMUM TRACTIVE EFFORT

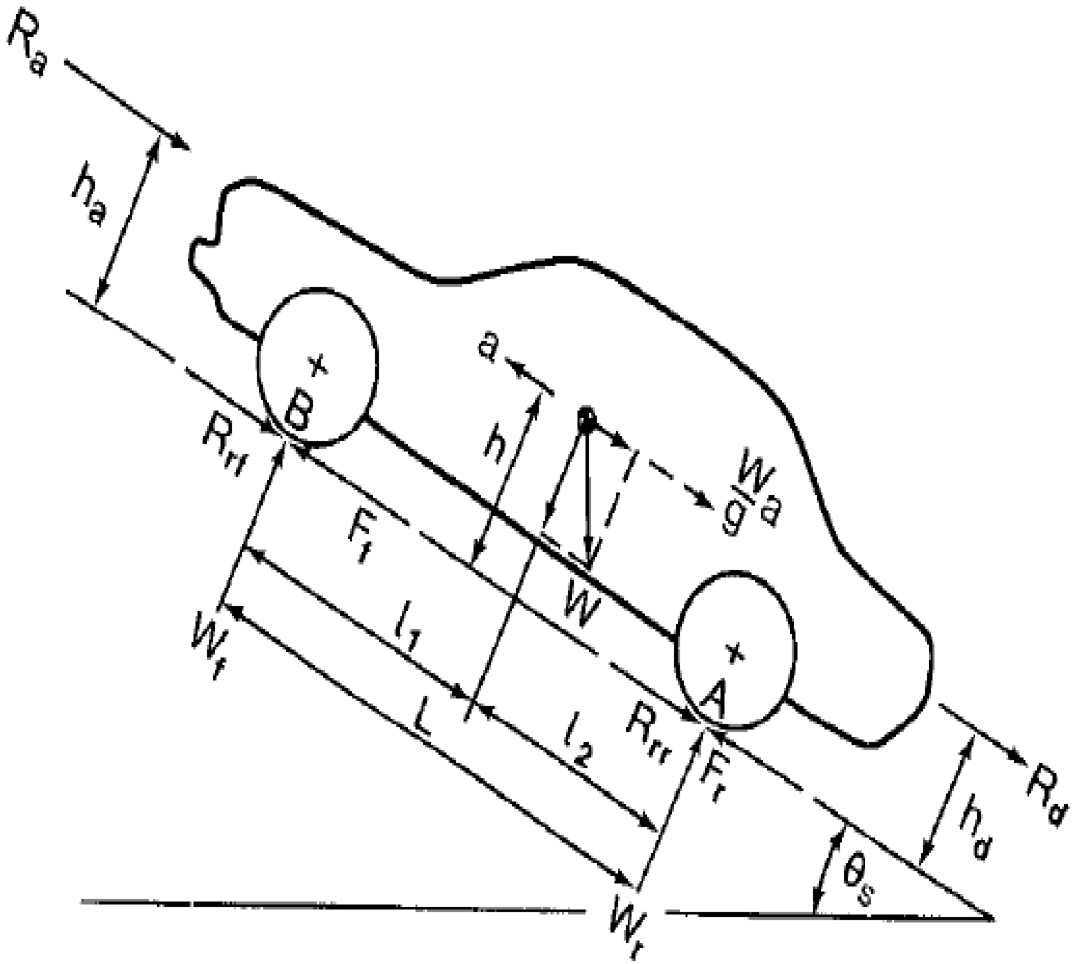


Fig 1: Force acting on a Two-axle vehicle

The equation of motion along the longitudinal axis x of the vehicle is expressed by

$$m\frac{d^2x}{dt^2} = \frac{W}{g}a = F_f + F_r - R_a - R_{rf} - R_{rr} - R_d - R_g \qquad \text{--------(1)}$$

By introducing the concept of inertia force, the above equation may be rewritten as

$$F_f + F_r - \left(R_a + R_{rf} + R_{rr} + R_d + R_g + \frac{aW}{g}\right) = 0$$

------(2)

or

$$F = R_a + R_r + R_d + R_g + \frac{aW}{g}$$

where F is the total tractive effort and R_r is the total rolling resistance of the
Vehicle.

- To predict the maximum tractive effort that the tire-ground contact can support, the normal loads on the axles have to be determined.
- They can be computed readily by summation of the moments about points A and B shown in Fig.1.
- By summing moments about A, the normal load on the front axle W_f can be determined:

$$W_f = \frac{Wl_2 \cos \theta_s - R_a h_a - haW/g - R_d h_d \mp Wh \sin \theta_s}{L}$$ -------(3)

When the vehicle is climbing up a hill, the negative sign is used for the term Wh $\sin\theta_s$.

Similarly, the normal load on the rear axle can be determined by summing moments about B:

$$W_r = \frac{Wl_1 \cos \theta_s + R_a h_a + haW/g + R_d h_d \pm Wh \sin \theta_s}{L}$$ ----(4)

In the above expression, the positive sign is used for the term Wh $\sin\theta_s$, when the vehicle is climbing up a hill.

For small angles of slope, cosθ, is approximately equal to 1.

For passenger cars, the height of the point of application of the aerodynamic resistance h_a, and that of the drawbar hitch h_d, may be assumed to be near the height of the center of gravity h.

With these simplifications and assumptions, Eqs.3 and 4 may be rewritten as

$$W_f = \frac{l_2}{L} W - \frac{h}{L}\left(R_a + \frac{aW}{g} + R_d \pm W \sin \theta_s\right) \quad \text{-------(5)}$$

$$W_r = \frac{l_1}{L} W + \frac{h}{L}\left(R_a + \frac{aW}{g} + R_d \pm W \sin \theta_s\right) \quad \text{-----------(6)}$$

Substituting Eq.2 into the above equations, one obtains

$$W_f = \frac{l_2}{L} W - \frac{h}{L} (F - R_r) \quad \text{-------(7)}$$

and

$$W_r = \frac{l_1}{L} W + \frac{h}{L} (F - R_r) \quad \text{------------(8)}$$

- It should be noted that the first term on the right-hand side of each equation represents the static load on the axle when the vehicle is at rest on levelground.
- The second term on the right-hand side of each equation represents the dynamic component of the normal load or dynamic load transfer.

The maximum tractive effort that the tire-ground contact can support can be determined in terms of the coefficient of road adhesion μ and vehicle parameters.

Where;

$F_{max} = \mu W$

$R_r = f_r W$ (f_r is coefficient of rolling resistence)

For a rear-wheel-drive vehicle,

$$F_{max} = \mu W_r = \mu \left[\frac{l_1}{L} W + \frac{h}{L} (F_{max} - R_r) \right] \quad \text{-----(9)}$$

and

$$F_{max} = \frac{\mu W (l_1 - f_r h)/L}{1 - \mu h/L}$$

For a front wheel-drive vehicle,

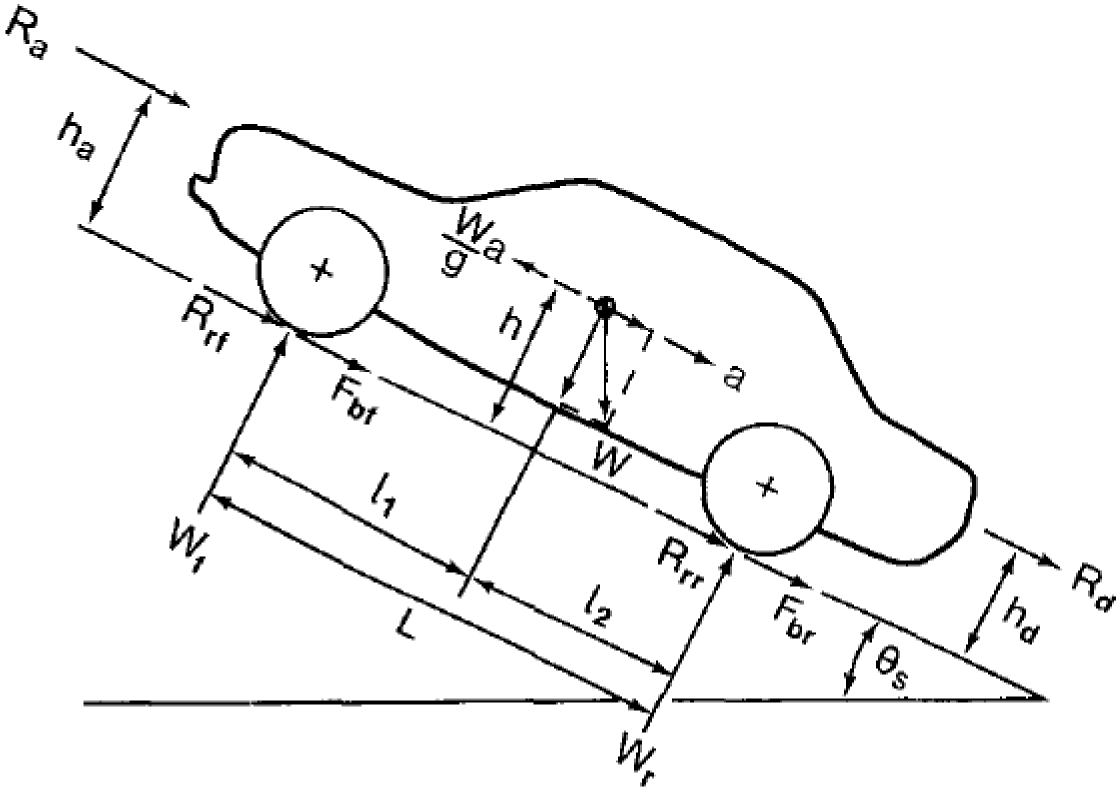
$$F_{max} = \mu W_f = \mu \left[\frac{l_2}{L} W - \frac{h}{L} (F_{max} - R_r) \right]$$

and ------(10)

$$F_{max} = \frac{\mu W (l_2 + f_r h)/L}{1 + \mu h/L}$$

Braking Characteristics of a Two-Axle Vehicle

The major external forces acting on a decelerating two-axle vehicle are shown in Fig.



Thus, the resultant retarding force F can be expressed by

$$F_{res} = F_b + f_r W \cos\theta_s + R_a \pm W \sin\theta_s + R_t \qquad \text{-----------(1)}$$

Where;

- f is the rolling resistance coefficient,
- W is the vehicle weight,
- θ_s is the angle of the slope with the horizontal,
- R_a is the aerodynamic resistance,
- R_t is the transmission resistance.

When the vehicle is moving uphill, the positive sign for the term W $\sin\Theta_s$, should be used. On a downhill grade, the negative sign should, however, be used.

Normally, the magnitude of the transmission resistance is small and can be neglected in braking performance calculations.

During braking, there is a load transfer from the rear axle to the front axle.

W_f and W_r can be expressed as

$$W_f = \frac{1}{L}\left[Wl_2 + h\left(\frac{W}{g}a - R_a \pm W\sin\theta_s\right)\right] \quad \text{-------(2)}$$

$$W_r = \frac{1}{L}\left[Wl_1 - h\left(\frac{W}{g}a - R_a \pm W\sin\theta_s\right)\right] \quad \text{--------(3)}$$

where a is the deceleration.

By considering the force equilibrium in the horizontal direction, the following relationship can be established:

$$F_b + f_r W = F_{bf} + F_{br} + f_r W = \frac{W}{g} a - R_a \pm W \sin \theta_s \qquad \text{------(4)}$$

Substituting Eq. 4 into Eqs. 2 and 3, the normal loads on the axles become

$$W_f = \frac{1}{L} [W l_2 + h(F_b + f_r W)] \qquad \text{---------(5)}$$

$$W_r = \frac{1}{L} [W l_1 - h(F_b + f_r W)] \qquad \text{------------(6)}$$

The maximum braking force that the tire-ground contact can support is determined by the normal load and the coefficient of road adhesion.

(assuming the maximum braking force of the vehicle $F_{bmax} = \mu W$

$$F_{bf\max} = \mu W_f = \frac{\mu W\,[l_2 + h(\mu + f_r)]}{L} \qquad \text{-------(7)}$$

$$F_{br\max} = \mu W_r = \frac{\mu W\,[l_1 - h(\mu + f_r)]}{L} \qquad \text{-------(8)}$$

Distribution of braking force

$$\frac{K_{bf}}{K_{br}} = \frac{F_{bf\max}}{F_{br\max}} = \frac{l_2 + h(\mu + f_r)}{l_1 - h(\mu + f_r)}$$

- where K_{bf} and K_{br} are the proportions of the total braking force on the front and rear axles, respectively, and are determined by the brake system design.

For instance, for a light truck with 68% of the static load on the rear axle

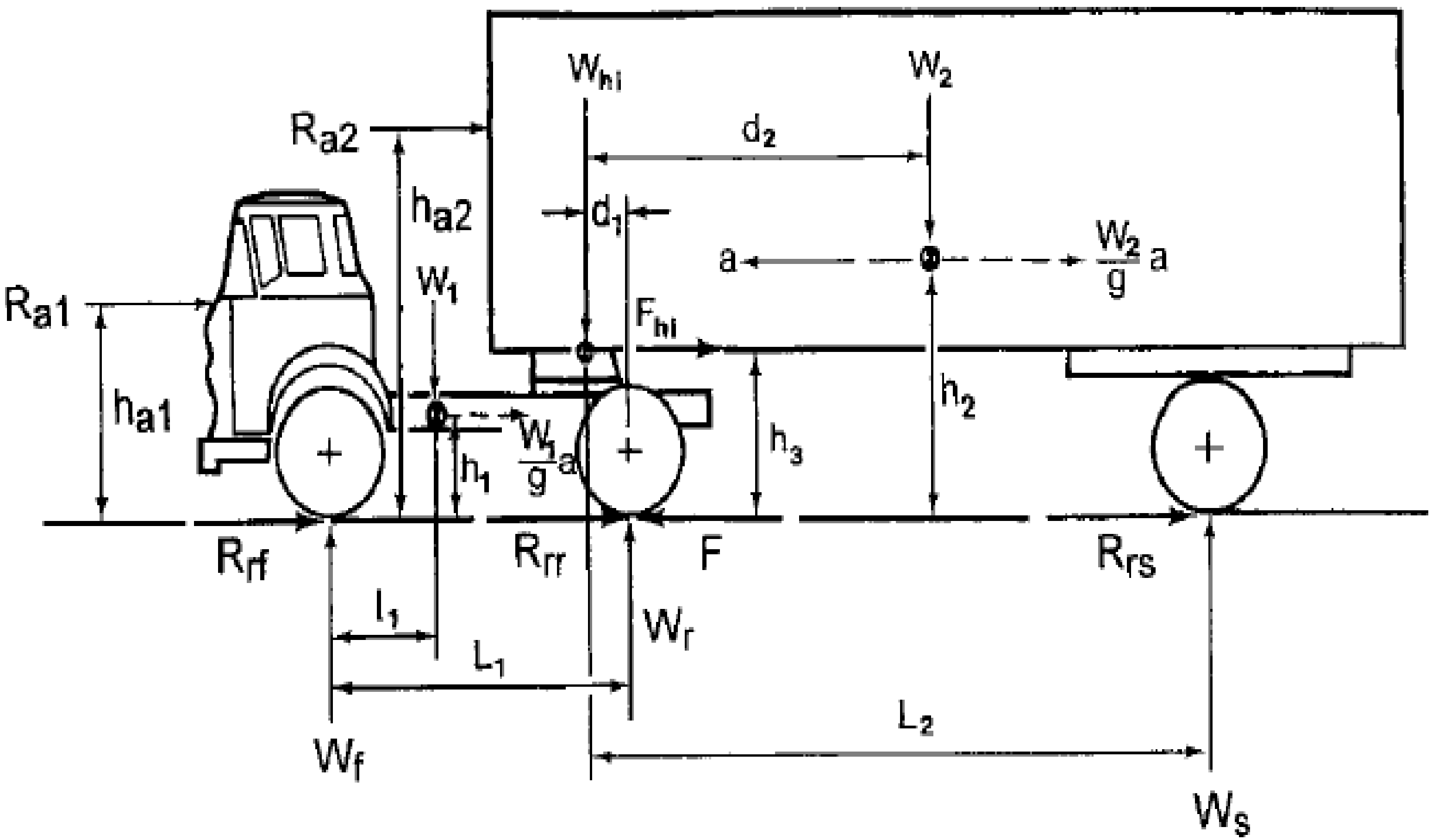
($l_2/L = 0.32$, $l_1/L = 0.68$),

$h/L = 0.18$,

$\mu = 0.85$, and $f_r = 0.01$,

$$\frac{K_{bf}}{K_{br}} = \frac{0.32 + 0.18(0.85 + 0.01)}{0.68 - 0.18(0.85 + 0.01)} = \frac{47}{53}$$

Tractor-semitrailer (Maximum Tractive effort)



- To compute the maximum tractive effort as determined by the nature of tire-road interaction,

- it is necessary to calculate the normal load on the tractor rear axle under operating conditions.

- This can be calculated by considering the tractor and the semitrailer as free bodies separately.

- By taking the semitrailer as a free body, the normal load on the semitrailer axle W_s

- And the vertical and horizontal loads at the hitch point W_{hi} and F_{hi} can be determined

The normal load on the semitrailer axle, for small angles of slope, is given by

$$W_s = \frac{W_2 d_2 + R_{a2} h_{a2} + h_2 a W_2 / g \pm W_2 h_2 \sin\theta_s - F_{hi} h_3}{L_2}$$

If $h_{a2} \simeq h_3 \simeq h_2$, the expression for W_s may be simplified as

$$W_s = \frac{d_2}{L_2} W_2 + \frac{h_2}{L_2}\left(R_{a2} + \frac{aW_2}{g} \pm W_2 \sin\theta_s - F_{hi}\right) \qquad \text{------(1)}$$

The longitudinal force at the hitch point is given by

$$F_{hi} = R_{a2} + \frac{aW_2}{g} \pm W_2 \sin\theta_s + f_r W_s \qquad \text{------(2)}$$

Substitute Eq.3 into Eq.2, the expression for W_s becomes

$$W_s = \frac{W_2 d_2}{L_2 + f_r h_2} \qquad \text{---------(3)}$$

and the load at the hitch point is given by

$$W_{hi} = W_2 - W_s = \left(1 - \frac{d_2}{L_2 + f_r h_2}\right) W_2 \qquad \text{---(4)}$$

$$= C_{hi} W_2$$

By taking the tractor as a free body and summing moments about the front tire–ground contact point, the normal load on the tractor rear axle W_r can be determined:

$$W_r = \frac{W_1 l_1 + R_{a1} h_{a1} + h_1 a W_1 / g \pm W_1 h_1 \sin\theta_s + F_{hi} h_3 + (L_1 - d_1) W_{hi}}{L_1}$$

If $h_{a1} \cong h_3 \cong h_1$, the expression for W_r may be simplified as

$$W_r = \frac{W_1 l_1 + (R_{a1} + a W_1 / g \pm W_1 \sin\theta_s + F_{hi}) h_1 + (L_1 - d_1) W_{hi}}{L_1} \quad \text{--(5)}$$

By equating the forces acting on the tractor in the longitudinal direction, the following expression for the required tractive effort F can be obtained:

$$F = R_{a1} + \frac{a W_1}{g} \pm W_1 \sin\theta_s + f_r (W_1 + W_{hi}) + F_{hi} \quad \text{------(6)}$$

From Eqs. 5 and 6, the maximum tractive effort that the tire-ground contact can support with the tractor rear axle driven can be expressed by

$$F_{\max} = \mu W_r = \frac{\mu[l_1 W_1 - h_1 f_r (W_1 + W_{hi}) + (L_1 - d_1) W_{hi}]/L_1}{1 - \mu h_1 / L_1}$$

Substitution of Eq. 4 into the above equation yields

$$F_{\max} = \frac{\mu[l_1 W_1 - h_1 f_r (W_1 + C_{hi} W_2) + (L_1 - d_1) C_{hi} W_2]/L_1}{1 - \mu h_1 / L_1}$$

The maximum tractive effort as determined by the nature of the tire-road interaction imposes a fundamental limit on the vehicle performance characteristics, including maximum speed, acceleration, gradability, and drawbar
pull.

Aerodynamic resistance

In practice, the aerodynamic resistance is usually expressed in the following form:

$$R_a = \frac{\rho}{2} C_D A_f V_r^2$$

Where

p is the mass density of the air,

C_D is the coefficient of aerodynamic resistance

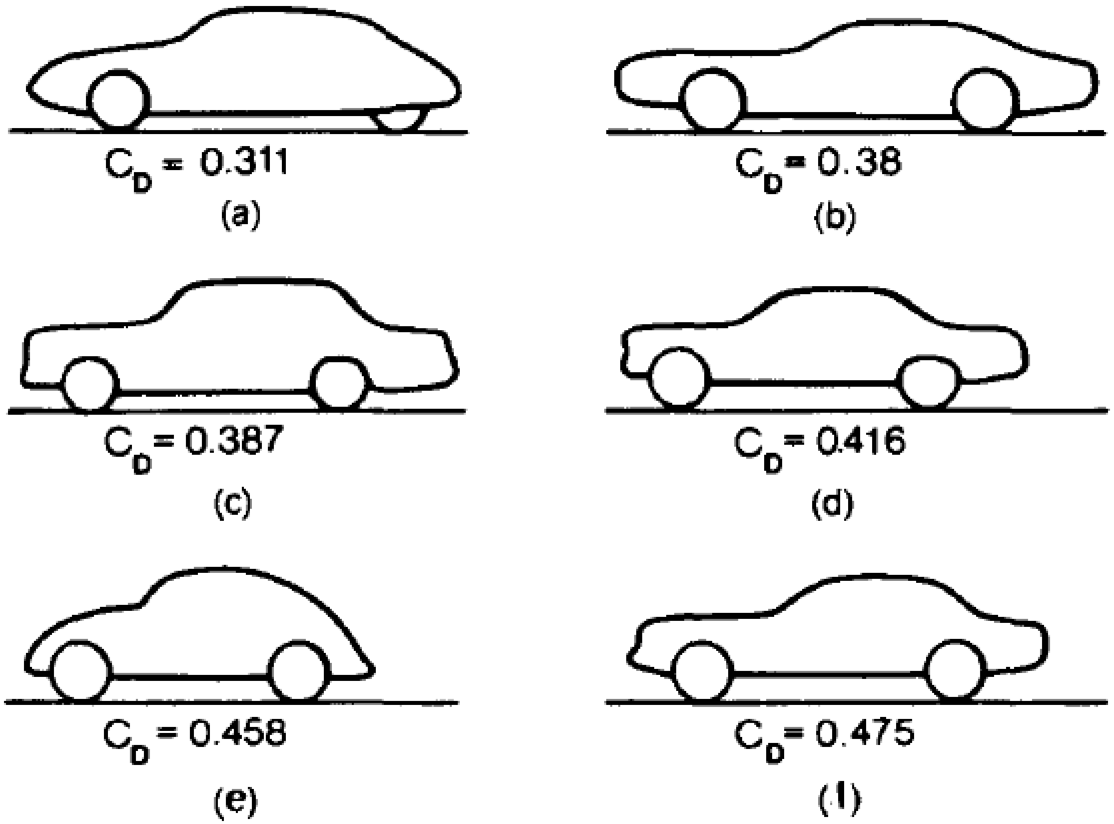
A_f is a characteristic area of the vehicle

V_r is the speed of the vehicle relative to the wind.

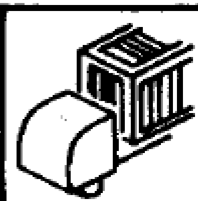
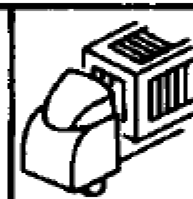
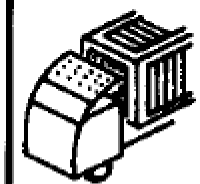
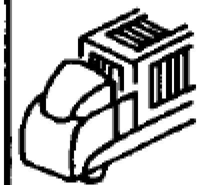
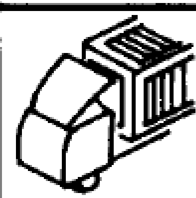
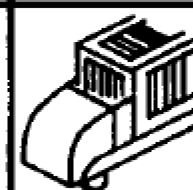
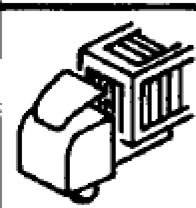
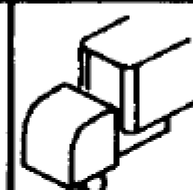
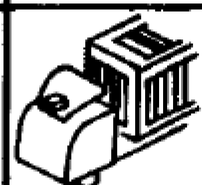
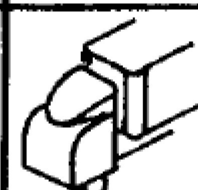
Based on data collected,
for passenger cars with mass in the range of 800-2000 kg (or 1760-4400 lb in weight), the relationship between the frontal area and the vehicle mass may be approximately expressed by

$$A_f = 1.6 + 0.00056\,(m_v - 765)$$

The influence of shape variations on the aerodynamic resistance coefficient of a passenger car is shown in Fig.



Aerodynamic resistance coefficient for passenger cars. (a) Citroen DS 19. (b) Oldsmobile Toronado. (c) Mercedes-Benz 300 SE. (d) Ford Falcon Futura. (e) VW 1200. (f) Ford Mustang.

		C_D			C_D
1		0.863	6		0.656
2		0.663	7		0.629
3		0.660	8		0.820
4		0.657	9		0.673
5		0.668	10		0.568

Acceleration Time and Distance

- To determined the net thrust of the vehicle as a function of speed, one can then compute the acceleration of the vehicle using Newton's second law.

- It should be noted, however, that the translational motion of the vehicle is coupled to the rotational motion of the components connected with the wheels, including the engine and the driveline.

- Any change of translational speed of the vehicle will therefore be accompanied by a corresponding change of the rotational speed of the components coupled with the wheels.

To take into account the effect of the inertia of the rotating parts on vehicle acceleration characteristics, a mass factor y_m is introduced into the following equation for calculating vehicle acceleration a:

$$F - \sum R = F_{\text{net}} = \gamma_m m a$$

where m is the vehicle mass.

y_m can be determined from the moments of inertia of the rotating parts by

$$\gamma_m = 1 + \frac{\sum I_w}{mr^2} + \frac{\sum I_1 \xi_1^2}{mr^2} + \frac{\sum I_2 \xi_2^2}{mr^2} + \ldots + \frac{\sum I_n \xi_n^2}{mr^2}$$

Where;

I_W is the mass moment of inertia of the wheel, I_1 , I_2 . . . I_n are the mass moments of inertia of the rotating components connected with the driveline having gear ratios ξ_1, ξ_2. . . ξ_n respectively, with reference to the driven wheel,

And r is the rolling radius of the wheel.

For passenger cars, the mass factor y_m may be calculated using the following empirical relation:

$$\gamma_m = 1.04 + 0.0025\xi_o^2$$

The first term on the right-hand side of the above equation represents the contribution of the rotating inertia of the wheels.

The second term represents the contribution of the inertia of the components rotating at the equivalent engine speed with the overall gear reduction ratio ξ_o with respect to the driven wheel.

In the evaluation of vehicle acceleration characteristics, time-speed and time-distance relationships are of prime interest. These relationships can be derived using the equation of motion of the vehicle in a differential form:

$$\gamma_m m \frac{dV}{dt} = F - \sum R = F_{\text{net}}$$

and

$$dt = \frac{\gamma_m m dV}{F_{\text{net}}}$$

the net tractive effort F_{net} available for accelerating the vehicle is a function of vehicle speed:

$$F_{\text{net}} = f(V)$$

This makes the expression relating the time and speed of the following form not integrable by analytic methods:

$$t = \gamma_m m \int_{V_1}^{V_2} \frac{dV}{f(V)}$$

The distance S that the vehicle travels during an acceleration period from speed V_1 to V_2 can be calculated by integrating the following equation:

$$S = \int_{V_1}^{V_2} \frac{V dV}{F_{\text{net}}/\gamma_m m} = \gamma_m m \int_{V_1}^{V_2} \frac{V dV}{f(V)}$$

Relationship between Tractive effort and power transmission

$$F = \frac{M_e \xi_o \eta_t}{r}$$

Where;

M_e is the engine output torque,

ξ_O is the overall reduction ratio of the transmission (including both the gearbox and drive axle gear ratios),

η_t is the overall transmission efficiency,

and r is the radius of the tire.

The relationship between vehicle speed and engine speed is given by

$$V = \frac{n_e r}{\xi_o}(1 - i)$$

where n_e is the engine speed and i is the slip of the vehicle running gear.

For a road vehicle, the slip is usually assumed to be 2-5% under normal operating conditions.

. A vehicle weighs 21.24 kN (4775 lb), including the four road wheels. Each of the wheels has a rolling radius of 33 cm (13 in.) and a radius of gyration of 25.4 cm (10 in.), and weighs 244.6 N (55 lb). The engine develops a torque of 325 N · m (240 lb · ft) at 3500 rpm. The equivalent mass of moment of inertia of the parts rotating at engine speed is 0.733 kg · m^2 (0.54 slug · ft^2). The transmission efficiency is 85%, and the total reduction ratio of the driveline in the third gear is 4.28 to 1. The vehicle has a frontal area of 1.86 m^2 (20 ft^2), and the aerodynamic drag coefficient is 0.38. The coefficient of rolling resistance is 0.02. Determine the acceleration of the vehicle on a level road under these conditions.

Solution

a) The mass factor y_m for the vehicle in the third gear can be calculated using

$$\gamma_m = 1 + \frac{\sum I_w + \sum I\xi^2}{mr^2}$$

$$= 1 + \frac{4 \times 1.61 + 0.733 \times 4.28^2}{2165 \times 0.33^2} = 1.084$$

Where;
W = m g

Radius of gyratio is

$$k = \sqrt{\frac{I}{M}}$$

b)The thrust of the vehicle F is determined using

$$F = \frac{M_e \xi_o \eta_t}{r} = 3583 \text{ N (806 lb)}$$

c) The vehicle speed V can be calculated using

$$V = \frac{n_e r}{\xi_o} = (1 - i)$$

Assume that $i = 3\%$; the vehicle speed V is

$$V = 98.7 \text{ km/h } (61.3 \text{ mph})$$

d) The total resistance of the vehicle is the sum of the aerodynamic resistance R_a and the rolling resistance R_r:

$$\sum R = R_a + R_r = 752 \text{ N } (169 \text{ lb})$$

Where;
$R_a = 1/2 * C_D \rho A^2$

$R_r = f_r W$

e) The acceleration a of the vehicle can be determined using

$$a = \frac{F - \sum R}{\gamma_m m} = 1.2\ \text{m/s}^2\ (3.94\ \text{ft/s}^2)$$

Reference

1) J.Y. Wong. Theory of ground vehicles. John Wiley & Sons, 2001.