CHAPTER 8. BOUNDARY LAYER CONCEPTS

8.1 INTRODUCTION

- A boundary layer is a very narrow region of flow near a boundary ~~for~~ in which, for a fluid flow with large Reynolds number, all the viscous effects can be considered to be confined.
- The flow outside the boundary layer can be considered to be ~~ideal~~ ideal fluid flow.
- Fig. 8.1 shows the flow past a sharp-edged smooth plate. The layer in which the flow is retarded, and consequently the local velocity u at any y above the plate is less than U, is the boundary layer.
- A typical velocity distribution in a boundary layer is shown in Fig 8.2.

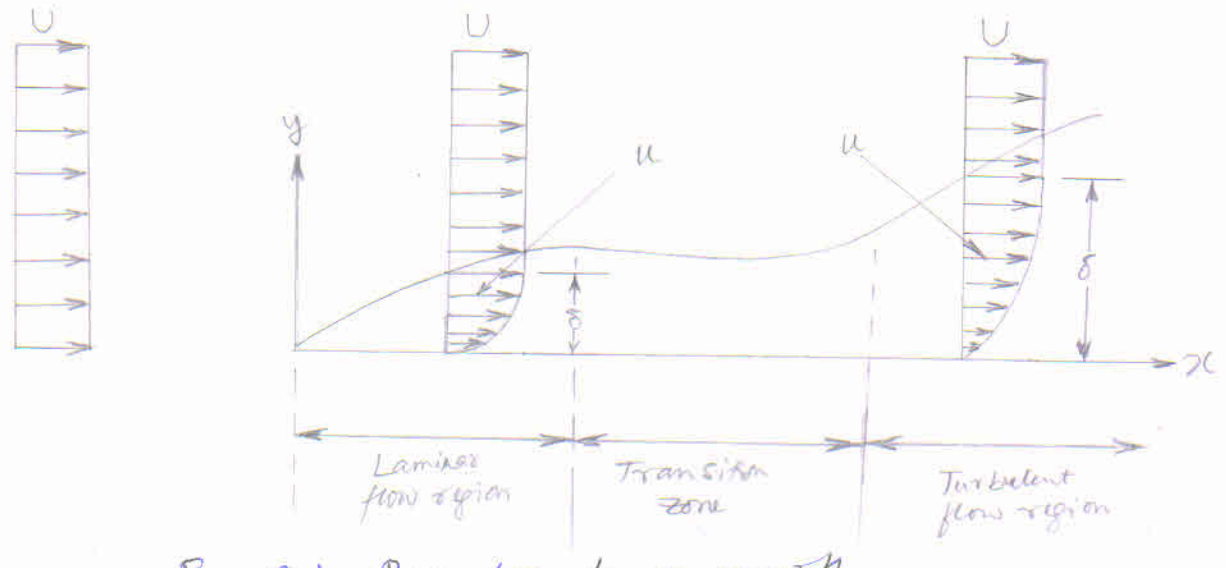


Fig. 8.1 Boundary layer growth

- As u reaches the free stream velocity U asymptotically, the boundary layer thickness (δ) is defined as the value of y at which ~~$u = 0.99$~~ $u = 0.99U$.

- The local Reynolds number is

$$Re_x = \frac{Ux}{\nu} = \frac{\rho U x}{\mu}$$

where ν = kinematic viscosity of the fluid
μ = dynamic " " "

- Initially the boundary layer will be laminar. But around a value of $Re_x = 5 \times 10^5$, the flow in the boundary layer will undergo a transition phase, and will soon become turbulent.
- A boundary layer in which the flow is turbulent is called a turbulent boundary layer.
- In a laminar boundary layer, the velocity distribution (i.e. variation of u with y) is parabolic, while it is logarithmic in a turbulent boundary layer.

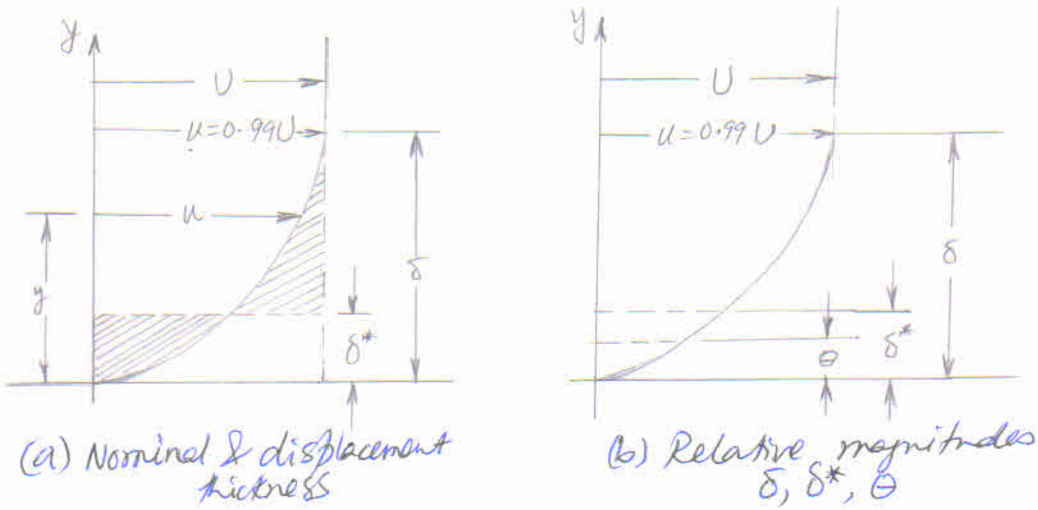


Fig 8.2 Boundary layer thickness

- In boundary layer theory, the following thicknesses of the boundary layer are defined and used.

(a) Nominal thickness (δ)

It is the thickness, measured from the boundary at which the x-component of the velocity attains 99% of the free stream velocity U, i.e. the value of y at which $u = 0.99U$.

(b) Displacement thickness (δ^*)

The Displacement thickness is defined as the distance by which the external potential flow is displaced outwards due to the decrease in velocity in the boundary layer.

{Potential flow is characterized by an irrotational velocity field}

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy \quad \text{------} (8.1)$$

(c) Momentum thickness (θ)

This is defined as the loss of momentum in the boundary layer as compared with that of potential flow.

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \quad \text{------} (8.2)$$

(d) Energy thickness (θ^*)

This is defined as $\theta^* = \int_0^{\delta} \frac{u}{U}\left(1 - \left(\frac{u}{U}\right)^2\right) dy$

$\text{------} (8.3)$

(e) <u>Shape factor (H)</u>

The <u>shape factor</u> of the boundary layer is defined as the ratio

$$H = \frac{\delta^*}{\theta} = \frac{\text{Displacement thickness}}{\text{Momentum thickness}} \quad (8.4)$$

8.2 BOUNDARY CONDITIONS

For a laminar boundary layer, the boundary conditions are:

(a) At the wall $(y=0)$: $u=0$ and $v=0$
(b) At the outer edge $(y=\delta)$: $u=U$
(c) Shear stress at the wall, $\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$

The flow over a flat plate which is described here is a particular case in which U = constant, or the pressure gradient $\frac{dp}{dx} = 0$. This case is also known as <u>zero pressure gradient flow</u>.

The boundary layer thickness (δ) and the local shear stress (τ_0), are functions of x.

8.3 LAMINAR BOUNDARY LAYER

For laminar flow over a flat plate, Blasius solved the basic boundary layer equations and obtained an analytical solution which have been verified experimentally to be remarkably accurate.

The <u>classic Blasius solution</u> for the laminar boundary layer is

$$\boxed{\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}} \quad \text{----} (8.5)$$

where $Re_x = \frac{Ux}{\nu}$

By defining $\tau_0 = C_f \frac{\rho U^2}{2}$

where C_f = local shear stress coefficient,

we have $$\boxed{C_f = \frac{0.664}{\sqrt{Re_x}}} \quad \text{----} (8.6)$$

If the total drag force <u>on one side</u> of ~~the~~ a plate of length L and width B is defined as

$$F_D = B\int_0^L \tau_0\, dx = C_{Df}\,(L \cdot B)\,\frac{\rho U^2}{2}$$ L ≡ length, B ≡ width

then $$\boxed{C_{Df} = \frac{1.328}{\sqrt{Re_L}}} \quad \text{----} (8.7)$$

where $Re_L = \frac{UL}{\nu}$, and C_{Df} = total frictional drag coefficient

<u>8.4</u> <u>TRANSITION FROM LAMINAR BOUNDARY LAYER</u>

- As the flow passes down the plate, i.e. as Re_x increases, the boundary layer thickness increases and it soon becomes unstable.
- Turbulence persists and grows in the boundary

layer at higher values of Re_x.

- It is generally believed that the transition from laminar to turbulent boundary layer takes place between $Re_x = 1.3 \times 10^5$ and 4×10^6, with the mean value of

$$Re_x = (Re_x)_{crit} = 5 \times 10^5$$

taken as the commonly accepted critical Reynolds number.

- In a flow past a long plate, the initial part in the boundary layer upto x_{crit} will be laminar, and then onwards the flow of the boundary layer will be turbulent.

8.5 TURBULENT BOUNDARY LAYER

- The turbulent boundary layer will have much steeper velocity gradients at the boundary than the laminar boundary layer.
- The velocity distribution in the turbulent b.l. is logarithmic, and can be conveniently expressed in the form of a power law

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

over a range of Reynolds number.

The power n can be 5 to 10, depending on the Reynolds number range.

- Next to the boundary, in a turbulent b.l. over a smooth ~~bed~~, there exists a thin layer called the laminar sublayer.
- For Re_x between 5×10^6 and 2×10^7, the velocity distribution can be expressed by

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{7}$$

- The turbulent b.l. characteristics found by experiments and analytical calculations, to be valid for $5 \times 10^5 < Re_x < 2 \times 10^7$, are

$$\delta / x = 0.377 / Re_x^{1/5}$$

$$c_f = 0.059 / Re_x^{1/5}$$

$$C_{Df} = 0.074 / Re_L^{1/5}$$

The above formulae assume the boundary layer to be turbulent from $x=0$.
To account for the initial laminar ~~sublayer~~ b.l., ~~C_D~~ C_{Df} can be calculated by

$$C_{Df} = \left(0.074 / Re_L^{1/5}\right) - \left(1700 / Re_L\right)$$

- For higher Reynolds numbers ($10^7 < Re_x < 10^9$), the logarithmic form of the velocity distribution in the turbulent b.l. is more appropriate. The boundary layer shear coefficients are expressed by the following formulae given by ~~Schlichting~~ Schlichting:

$$C_f = 0.370/(\log Re_x)^{2.58}$$
$$C_{Df} = 0.455/(\log Re_L)^{2.58}$$

The boundary layer thickness is estimated by

$$\delta/x = 0.22/Re_x^{1/6}$$

If correction for initial laminar b.l. is applied, then

$$C_{Df} = 0.455/(\log Re_L)^{2.58} - 1700/Re_L$$

The term $1700/Re_L$ is so small that omitting it does not cause any appreciable error.

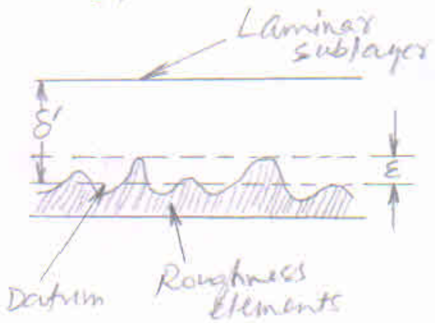
8.5 LAMINAR SUBLAYER

- The laminar sublayer is usually very thin and its thickness has been found from experiments (δ').
- If the roughness magnitude of a surface (ε) is very small compared to δ', ie.

$$\varepsilon << \delta'$$

then such a surface is said ~~to by to be~~ to be hydrodynamically smooth.
Roughness does not have any influence in such flows while the viscous effects predominate.
- Usually $\varepsilon/\delta' < 0.25$ is taken as the criterion for hydrodynamically smooth surface. (Fig. 8.3)

(a) Smooth boundary



(b) Rough boundary

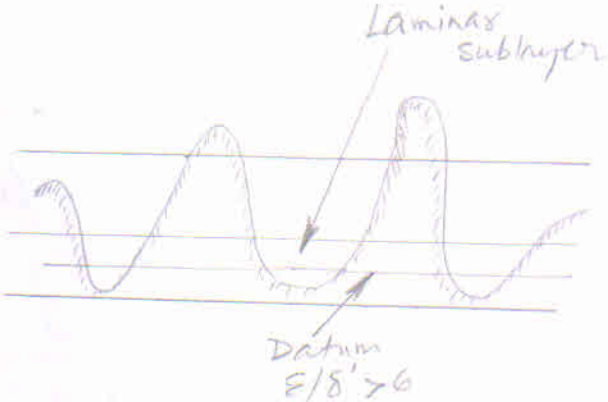


Fig. 8.3 Smooth & rough surfaces

- If the Laminar sublayer thickness (δ') is very small compared to roughness height (ε),

  i.e. $\varepsilon \gg \delta'$

  in such flows viscous effects are not important and the boundary is said to be hydrodynamically rough.

  Usually $\varepsilon / \delta' > 6$ is taken to be the criterion for hydrodynamically rough boundaries.

- In the region $0.25 < \varepsilon/\delta' < 6$, the boundary is in the transition regime, and both viscosity and roughness control the flow.

8.7 BOUNDARY LAYER SEPARATION

8.7.1 Separation phenomenon

- The flow past a flat plate held parallel to the flow is a case of b.l. with zero pressure gradient.

- Flows in converging boundaries are examples of favorable pressure gradient, and flows in diverging conduits or diverging boundaries are examples of adverse pressure gradient flows.
- In adverse pressure gradient boundary layer flow, the b.l. may at some section leave the boundary. This is called separation, and downstream of the separation section, turbulent eddies exist, and this disturbed region is called the wake.
- Separation can take place both in laminar and turbulent b.l.
- The location of the separation section on the surface of a body and the size of the wake have important bearing on the total drag force experienced by the body.
- At the separation point, the shear stress is zero, and the velocity gradient $\frac{\partial u}{\partial y} = 0$.

8.7.2 Control of separation

- Separation of flow from the boundary leads to ~~a~~ inefficiency of the flow unit.
- In lifting surfaces such as airfoils, it may cause reduction of lift, and even stalling. Diffusers, conduit transitions, pump

and turbine blades and airfoils, are some common flow units where separation may impair performance.

- Common procedures to control separation are based on the following methodologies (Fig. 8.4):

    * Streamlining of blunt body shapes
    * Fluid injection into the b.l
    * Suction of fluid from the b.l
    * Creating a motion of the boundary wall.

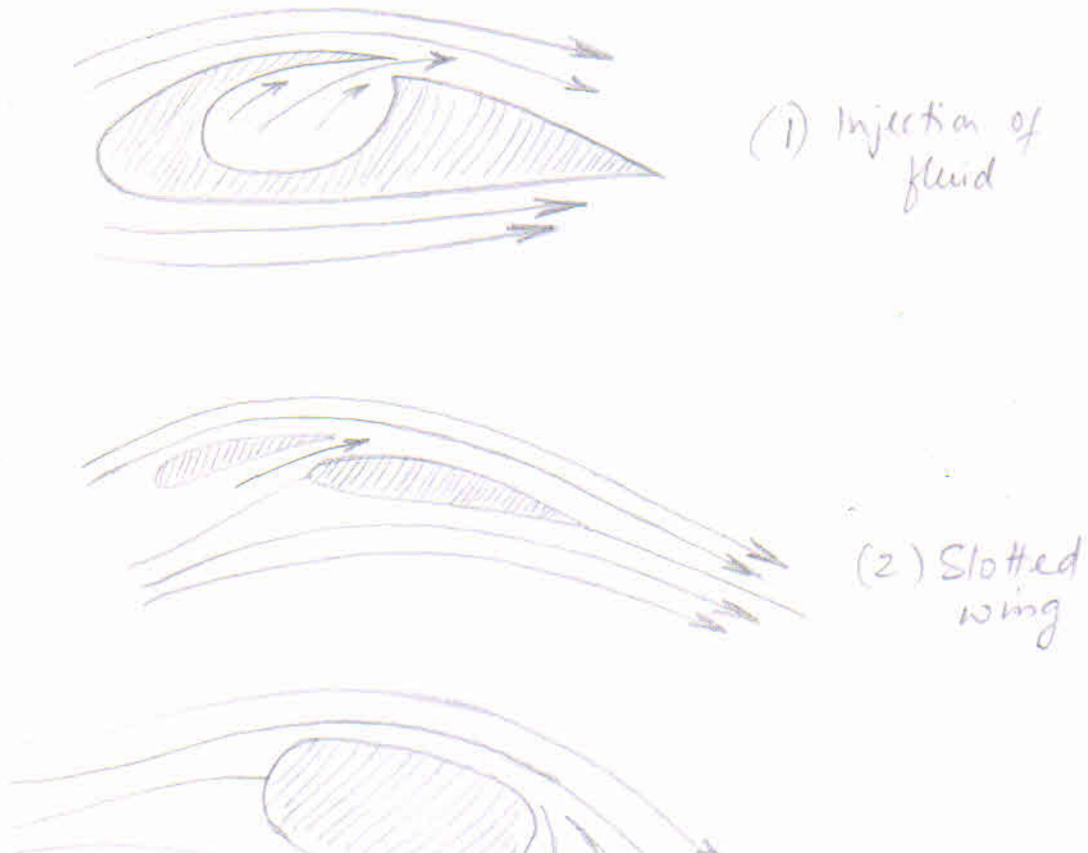


Fig. 8.4 Different arrangements for boundary layer control